## Question 1

1. I don’t think this is taught any more.
2. **A** needs to be a square matrix so after the transpose its dimension can still match. And the diagonal elements should be 0, since we want aii = - aii and the only value that is possible for this is 0.
3. Do the singular value decomposition by calculating AT \* A = [[14,11],[11,14]] and the eigenvalues are 25 and 3. By taking the square root of both we get the singular values which are 5 and sqrt(3). So the l\_2 norm is 5, i.e. the largest singular value.

## Question 2

a)

i)

1.We first show d(x,x) = 0

This follows easily from the definition.

2.We then show that d(x,y) > 0 if x != y, this also follows easily from the definition of absolute value.

3.We then show that d(x,y) = d(y,x)

d(x,y) = abs(x - y) = abs(-(y-x)) = abs(y-x) = d(y,x)

4. Finally we show that d(x,y) <= d(x,z) + d(z,y)

d(x,y) = abs(x-y) = abs((x-z) + (z-y)) <= abs(x-z) + abs(z-y) = d(x,z) + d(z,y)

ii)

And to prove that a sequence is Cauchy, we can prove a sequence is convergent. And as we can see 1/n tends to 0 when n tends to 0.

iii)

And to show that a metric space is complete, we show that for arbitrary Cauchy sequence, it is always convergent and the limit lies in the space, i.e. (0,1].

b) This is quite a strange question. Initially I thought it must be that one of the matrix is not positive semi-definite thus one of them can’t be solved by Cholesky decomposition. However, it turns out that both matrices are symmetric and all of their diagonal elements are positive and their largest element in absolute value are in diagonal. And after enter the two matrices into Wolfram Alpha, it turns out that both of them are positive semi-definite. Later I realize that using Cholesky decomposition to calculate the first matrix will produces fractions in the lower triangular matrix **L**, while this is not the case for the second one, i.e. the second one is much easier to calculate. So I guess the answer that the question want is to apply Cholesky Decomposition to the second matrix. And after doing just that, we obtain the **L** which is [[2,0,0],[-1,3,0],[1,0,1]], and after forward substitution and then backward substitution, we obtain the vector **x** which is [1,2,3]T.

c) It is linear dependent since **a2**=- **a1** - **a3**

## Question 3

1. i) the first one is saddle point at (0,0) ii) the second one is don’t know iii) the third one is a minimum
2. (ABC)^T = (A(BC))^T = (BC)^T A = C^T B^T A^T the proof for the second part can be found in [here](https://math.stackexchange.com/questions/464151/proving-distributivity-of-matrix-multiplication).
3. (A^T A)^T = A^T (A^T)^T = A^T A (A A^T)^T = (A^T)^T A^T = A A^T

## Question 4

1. A^-1 = [[B^-1 0],[d^T, 1]] where d is a m-vector and each element is the negative of the sum of all the column elements in B^-1.
2. (A+B)^2 = (A+B)(A+B) = A(A+B) + B(A+B) = A^2 + AB + BA + B^2 so we need to have AB = BA and both A and B needs to be square matrices having the same dimension otherwise we can’t do (A+B) \* (A+B) in the first place. The condition for (A+B)(A-B) follows similarly, (A+B)(A-B) = A(A-B) + B(A-B) = A^2 -AB +BA -B^2.
3. **AB** = [[c+a,1+ad],[bc+1, b+d]] and **BA** =[[c+b,1+ac],[bd+1, a+d]] and if we make these two matrices equal we get a = b and c = d.